International Journal of Mechanical and Thermal Engineering

E-ISSN: 2707-8051
P-ISSN: 2707-8043
IJMTE 2024; 5(2): 01-06
www.mechanicaljournals.com/ijmte
Received: 02-04-2024
Accepted: 06-05-2024

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Analysis of unsymmetrical cracks emanating from a hole placed centrally in a thin plate using fem

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Abstract
Center-holed plate, or a plate with a hole in the centre of a front face, the stress intensity factors of unsymmetrical cracks that started from hole edges under tension were determined. The center-cracked plate's crack length was compared to the sum of the diameter of the hole and the lengths of the cracks in the case of the holed plate in order to determine the stress intensity in the case of the holed plate with cracks. In the current study, counterplots of the stress intensity factor K_I are used to better understand how the stress intensity factor varies with respect to crack length, hole diameter and biaxial load factor.

Keywords: Fracture mechanics, stress intensity factor, unsymmetrical cracks, finite element method

Introduction

Holes and discontinuities are inevitable in structures. Cracks can emanate from these holes and can be inimical to structural integrity. Holes emanating from cracks has been an area of research for past many years [1]. Was the one who first calculated the value of stress intensity factor within a finite width plate for hole edge crack [2]. Studied the cracks emanating from a circular hole under biaxial loading [3]. Obtained the values of SIF of one hole-edge crack and also for the two unequal cracks at the two opposite side of hole-edge. Many times cracks can initiate unsymmetrically around the hole and as such it becomes important to analyze their dependency on various geometric factors. Stress Intensity factor, is the estimation of state of stress or stress intensity close to the crack-tip. K can be expressed in the form of applied stress near the crack tip of applied stress. According to the linear theory, stresses at the crack is infinity but plastic zone is always present within the crack-tip, limiting the stresses to a finite values. Isotropic linear elastic material produce $1/\sqrt{r}$ singularity close to the crack-tip. Magnitude of stress intensity factor depend upon geometry of sample, size, crack or notch location and loads on the materials.

The configuration of unsymmetrical cracks emanating from holes is so complex that analytical solution is not feasible very times. These kind of complex problems have been solved using numerical techniques like FEM, BEM, etc. Keeping in view the literature reviewed FEM is the most practical method to solve these kind of problem. There are various approaches for calculation of fracture parameters. Some of these approaches are the conformal mapping method was used by [4] rapid mesh refinement by [5, 6] element formulation by [7] finite element analysis by [8] minimum potential energy principle by [3] finite element program by presented by [9], super positioning technique by [10], body force method by [11], numerical analysis by [12], empirical formulation by [13], body force method by [14], convergence method of stress function by [15], analytical method by [16], digital image correlation by [17, 18], new approximation mapping function by [19], numerical method by [20] LAMMPS (Large-scale Atomic/Molecular Massively Parallel Simulator) by [21].

Materials and Methods

Aluminium is the world’s most abundant metal and is third most common element comprising 8% of earth’s crust. The versatility of aluminium makes it most widely used metal after steel. It is silvery white metal.

It is obtained from bauxite mineral. By the process of Bayer process bauxite is converted into aluminium oxide (alumina). With the help of Hall-Heroult process and electrolytic cell alumina is converted to aluminium metal.

Table 1: Material properties of Aluminium

Material	Young's Modulus (GPa)	Poisson's ratio (ν)	Density (kg/m^3)
Aluminium	71	0.33	2730

Numerical approach

For a thin plate, unsymmetrical cracks emanating from a circular hole, stress intensity factor were calculated using FEM software (ANSYS 2017) in APDL. For different conditions analysis was done on various crack orientations.

Finite Element Method (FEM)

Finite element method is a numerical technique for obtaining approximation solution to problems governing partial differential equations. These problems are boundary value problems. The problem or domain is divided into small parts called sub- domain or finite elements which are interconnected with each other at joints, called as nodes. Each node is having finite degree of freedom in the continuum with fixed properties. Group of elements is called mesh. The process of representing a component as a group of elements is called discretization.

Steps involved in FEM

Basically, there are three steps in which whole problem is solved.

1. **Preprocessing:** Discretization of problem and subdivision of domain.
2. **Processor:** Initial value generation and applying boundary conditions.
3. **Postprocessor:** Visualization and representation of results in graphs.

Specimen geometry and Assumptions

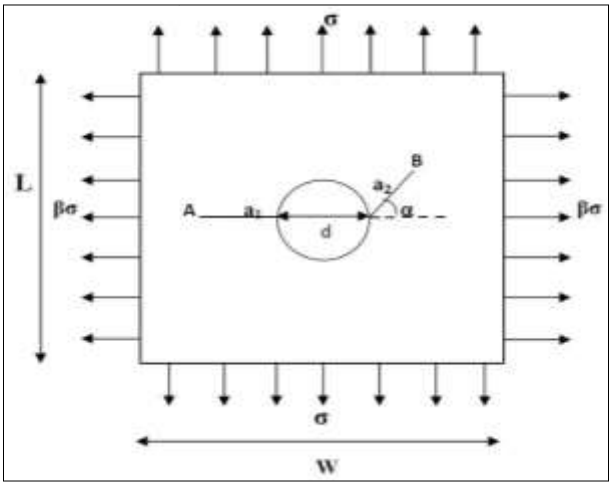

Fig 1: Specimen Geometry

Where,
L (length of specimen) = 10 mm
W (width of the specimen) = 10 mm
α= crack inclination angle (α= 0° to 60°)
σ = load applied in y-direction
βσ = load applied in x-direction
a_1 and a_2 = unsymmetrical crack length
A, B = Crack tip
β = biaxial load factor

For the present Aluminium plate, uniformly distributed load and plane stress condition with following dimensions was proposed. Stress intensity factor and T-stress were calculated for different configuration of cracks a_1 and a_2 which is presented in the table:

Table 2: Specification of specimen

S. No.	a_1 (mm)	a_2 (mm)	W (mm)	L (mm)	β	α (degrees)	d (mm)
1.	2	2	10	10	0.5	0° to 60°	2
2.	2	2	10	10	0 to 2	45°	2
3.	2	2	10	10	0.5	30°	1 to 3

The above geometries will be helpful in understanding the effect of geometrical parameters crack inclination angle (α), biaxility factor (β), and hole diameter (d) on SIF.

Validation

Analytical results are available for simple problems. For the complex problems particularly involving interacting cracks analytical solution are not available. In these cases numerical methods are used. However, to use them it becomes important to check the accuracy. In this section validation of Stress Intensity Factor (SIF) is obtained by FEM has been done with already existing analytical results.

Validation of mode I and mode II stress intensity factors under biaxial loading

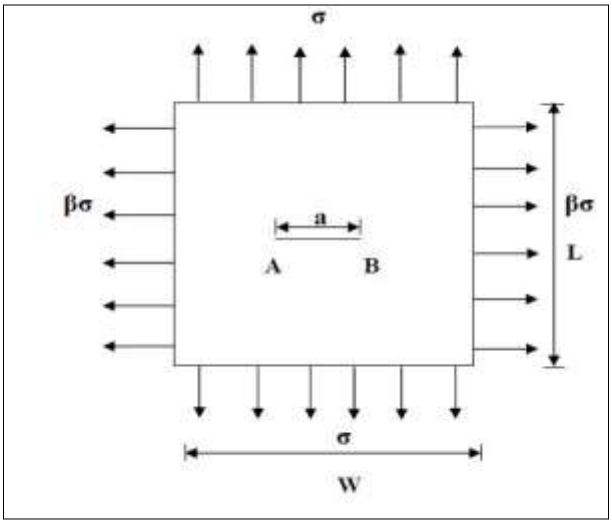

Fig 2: Single central cracked plate under biaxial loading

Where,
L= 10mm, length of the specimen
W= 10mm, width of the specimen
a= crack length in mm
σ= 200 MPa, load in y direction
βσ= 100 MPa, load in the x- direction
β = 0.5, biaxial load factor
α= inclination angle (0° to 60°)

Fig. 2 shows a plate with a central crack under biaxial loading. Results obtained by FEA were compared with theoretical results for various crack inclination angle (α) by [22] under biaxial loading. The numerical expression for rectangular plate with an inclined plate under biaxial loading is given by:

$K_{IA} = K_{I(0)} (\cos^2\alpha + \beta \sin^2 \alpha)$ (4.1)

$K_{IIA} = K_{I(0)} (\sin \alpha * \cos \alpha) (1-\beta)$ (4.2)

K_{IA}= Mode I stress intensity factor at crack tip A
K_{IIA}= Mode I stress intensity factor at crack tip A
$K_{I(0)}$= Mode I stress intensity factor at crack tip A at 0°
$K_{II(0)}$= Mode II stress intensity factor at crack tip A at 0°

Table 3 shows the error % for various cases of crack inclination angle α. It can be seen that the maximum error occurs at α=60° which can be considered safe looking at the advantages FEM has to offer.

$$\%\,Error = \frac{K_{FEM} - K_{Theo}}{K_{Theo}} \times 100$$

Table 3: Results of validation of K_I and K_{II} under biaxial loading

Inclination Angle (α)	$K_{IA\ (Theo)}$ (MPa√mm)	$K_{IA\ (FEM)}$ (MPa√mm)	% Error	$K_{IIA\ (Theo)}$ (MPa√mm)	$K_{IIA\ (FEM)}$ (MPa√mm)	% Error
15°	350.42	350.46	0.01	45.32	47.34	4.27
30°	317.24	320.94	1.16	78.49	81.45	3.77
45°	271.92	280.91	3.20	90.64	94.34	3.92
60°	226.60	236.45	4.34	78.49	81.52	3.86

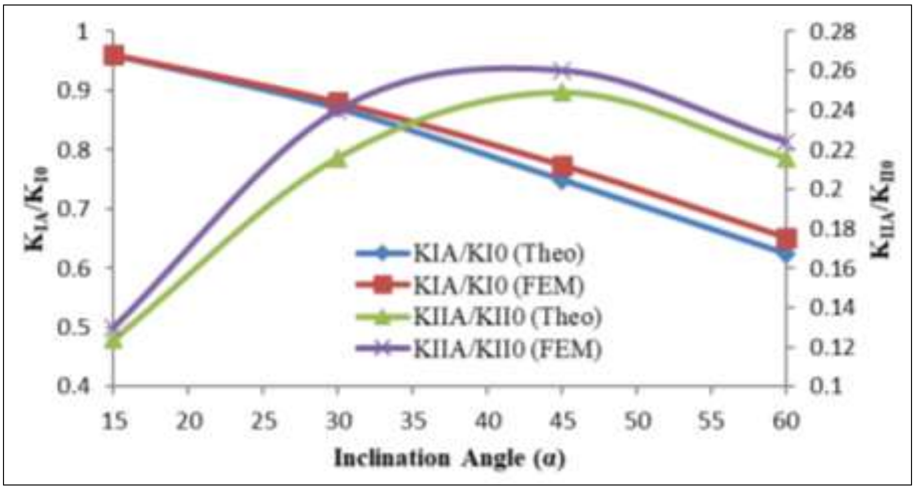

Fig 3: Variation of K_I and K_{II} SIF with crack inclination angle (α) at crack tip A

Fig. 3 shows the variation of mode I and mode II SIF with crack inclination angle (α) obtained by FEM in comparison to the analytical results. As can be seen from the graph the trend of variation is similar for both the numerical and analytical method.

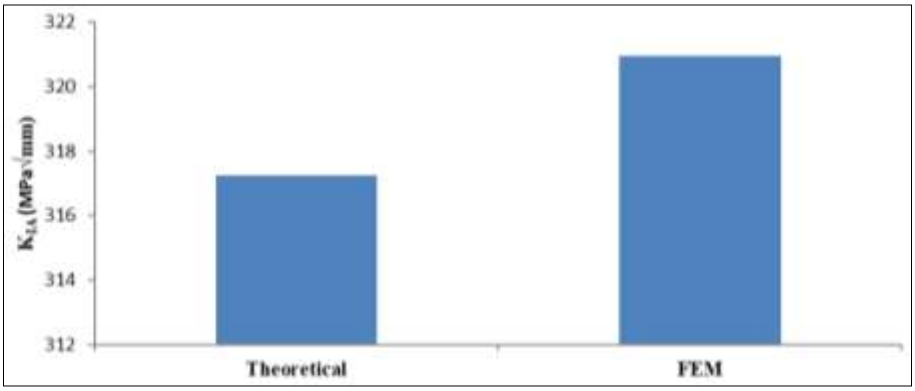

Fig 4: Comparison of results between FEM and theoretical stress intensity factor

Fig 4 shows theoretical and FEM solution of stress intensity factor of single central crack. At 30°, $K_{I(Theo)}$ is 317.24 MPa√mm and $K_{I(FEM)}$ is 320.94 MPa√mm amounting to % error of 1.16%.

Results and Discussion

Effect of crack inclination angle (α) on SIF of crack tip A and B: Fig. 5 shows the variation of both mode I and mode II SIF on the crack tip A and B with the crack inclination angle (α=0° to 60), keeping other parameters constant. It has been observed in the previous studies by Gope *et al.* (2014) that there is an interaction present among the neighbouring cracks. It can be observed that as the angle of inclination of one crack increases a strong interaction starts occurring between the two cracks.

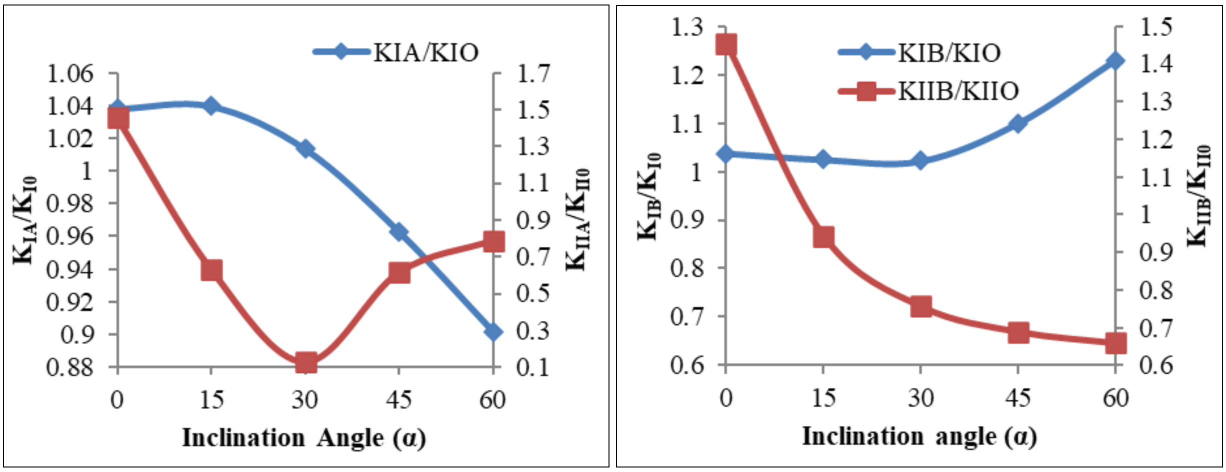

Fig 5: Variation of K_I and K_{II} SIF with crack inclination angle (α) of crack tip A and B respectively

Keeping both the trends in mind it seems that the structure is far safer when the crack is inclined rather than collinear cracks. Mode II becomes predominant failure mode for crack inclination angle greater than 30°, before which mode I is predominant. It can be concluded that greater the inclination angle better is for structural integrity. This reveals that just by changing the inclination of second crack (crack B), the stress intensity factor at crack tip of horizontal crack significantly affected.

Effect of biaxial load factor (β) on SIF for crack tip A and B: Fig. 6 shows the variation of both mode I and mode II SIF on the crack tip A with the variation of biaxial load factor (β)= 0.5 to 2. It can be observed from the graph that when β becomes greater than 1, mode II loading becomes predominant i.e. the cracks are under shearing action. At β= 2, mode II becomes 0.58 times mode I. This behavior is expected since β increases means that force in y-direction is greater and it would obviously induce shearing load on the crack faces and as such mode II would dominate.

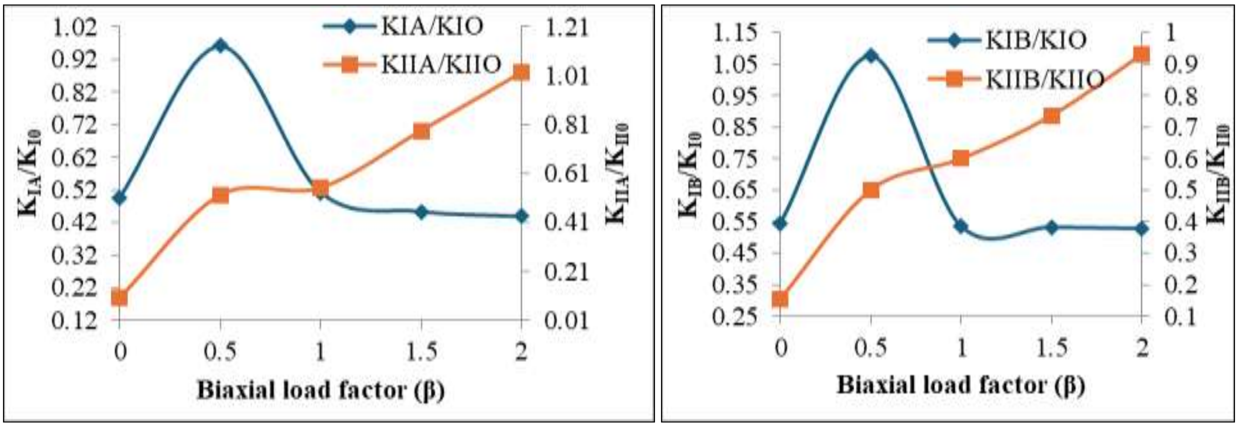

Fig 6: Variation of K_I and K_{II} SIF with biaxial load factor (β) of crack tip A and B respectively

It can be seen that when β becomes greater than 0.85 mode II loading becomes predominant i.e. the cracks are under shearing action at crack tip B. From the SIF behaviour of both the crack tips it can be concluded that as the biaxiality factor increases mode II loading becomes predominant. For biaxilality factor greater than 2, mode II SIF becomes greater than for uniaxial loading for both crack tip A and B. The shearing action is more pronounced in this case. It can be concluded that for cracks emanating from hole biaxial loading becomes critical for β>2.

Effect of variation of hole diameter on SIF on crack tip A and B: Fig. 7 shows the variation of both mode I and mode II SIF on the crack tip A with the variation of hole diameter (d= 1 to 3). The decrease in mode II SIF is seen to be phenomenal as diameter (d) of hole increases upto 2. After 2mm diameter, mode II SIF further increases significantly thus producing intensification effect.

It can be concluded that bigger holes would produce more SIF for cracks of similar length and hence hole diameter is going to play a critical role in determining the structural integrity. At d= 3 mm, mode I= 0.60 times mode II.

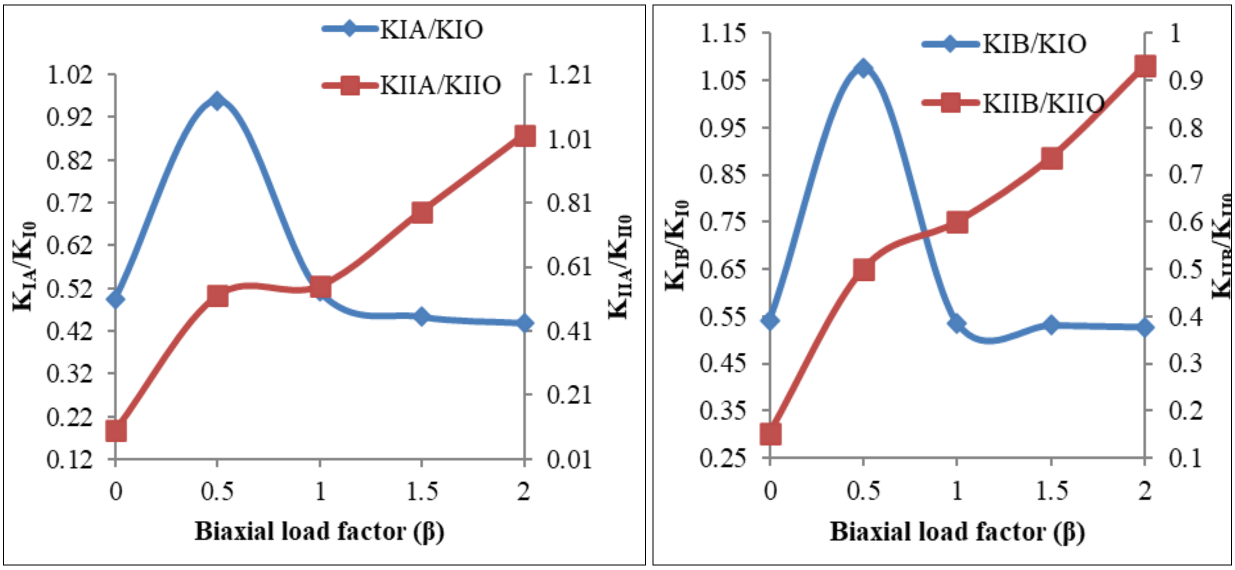

Fig 7: Variation of K_I and K_{II} SIF with hole diameter (d) of crack tip A and B respectively

At crack tip B, the hole diameter has greater impact as both stress intensity factor continuously increases. It can be concluded that hole diameter is going to play a critical role in determining structural integrity. After d= 2.5, mode II is predominating to mode I. Hence, mode II is 0.72 times mode I.

Conclusion

Various effects are observed with the variation of crack inclination angle, biaxial load factor, hole diameter and crack length ratio respectively.

1. With the variation of crack inclination angle, it seems that the structure is far safer when the crack is inclined rather than collinear cracks. It can also be concluded that greater the inclination angle better is for structural integrity.
2. With the variation of biaxial load factor, It can be seen that when β becomes greater than 1 for tip A and for tip B when β becomes greater than 0.85, mode II loading becomes predominant i.e. the cracks are under shearing action.
3. With the variation of hole diameter, For crack tip A and B, mode I SIF increases with increase in hole diameter (d). The mode II SIF is seen to be phenomenal and is seen to be further decreasing as diameter (d) of hole is increases upto 2, then it starts to increase for crack tip A whereas for crack tip B, mode II SIF increases significantly producing intensification effect.

Future Scope

In a nutshell it can be concluded that for assessing structural integrity of cracks emanating from holes various parameters have to be analyzed. Both shielding and intensification effect are present and there is a presence of mixed mode loading.

a) Fracture parameters such as J- integral, strain energy density can be calculated for various load factor
b) Life of a specimen including cracks can be predicted by different fracture laws such as Paris Law.
c) Unsymmetrical cracked problem under biaxial loading can be considered more easily using FEM with accuracy and efficiently.

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